

Exercise 33Calculate y' .

$$y = \ln |\sec 5x + \tan 5x|$$

SolutionCalculate y' by using the chain rule repeatedly.

$$\begin{aligned} y' &= \frac{d}{dx} \ln |\sec 5x + \tan 5x| \\ &= \frac{d}{dx} \ln \sqrt{(\sec 5x + \tan 5x)^2} \\ &= \frac{1}{\sqrt{(\sec 5x + \tan 5x)^2}} \cdot \frac{d}{dx} \sqrt{(\sec 5x + \tan 5x)^2} \\ &= \frac{1}{\sqrt{(\sec 5x + \tan 5x)^2}} \cdot \frac{1}{2} [(\sec 5x + \tan 5x)^2]^{-1/2} \cdot \frac{d}{dx} (\sec 5x + \tan 5x)^2 \\ &= \frac{1}{\sqrt{(\sec 5x + \tan 5x)^2}} \cdot \frac{1}{2} [(\sec 5x + \tan 5x)^2]^{-1/2} \cdot 2(\sec 5x + \tan 5x)^1 \cdot \frac{d}{dx} (\sec 5x + \tan 5x) \\ &= \frac{1}{\sqrt{(\sec 5x + \tan 5x)^2}} \cdot \frac{1}{\sqrt{(\sec 5x + \tan 5x)^2}} \cdot (\sec 5x + \tan 5x) \cdot \frac{d}{dx} (\sec 5x + \tan 5x) \\ &= \frac{1}{(\sec 5x + \tan 5x)^2} \cdot (\sec 5x + \tan 5x) \cdot \frac{d}{dx} (\sec 5x + \tan 5x) \\ &= \frac{1}{\sec 5x + \tan 5x} \cdot \frac{d}{dx} (\sec 5x + \tan 5x) \\ &= \frac{1}{\sec 5x + \tan 5x} \cdot \left[\frac{d}{dx} (\sec 5x) + \frac{d}{dx} (\tan 5x) \right] \\ &= \frac{1}{\sec 5x + \tan 5x} \cdot \left[(\sec 5x \tan 5x) \cdot \frac{d}{dx} (5x) + (\sec^2 5x) \cdot \frac{d}{dx} (5x) \right] \\ &= \frac{1}{\sec 5x + \tan 5x} \cdot [(\sec 5x \tan 5x) \cdot (5) + (\sec^2 5x) \cdot (5)] \\ &= \frac{1}{\sec 5x + \tan 5x} \cdot (5 \sec 5x \tan 5x + 5 \sec^2 5x) \\ &= \frac{1}{\sec 5x + \tan 5x} \cdot (5 \sec 5x)(\tan 5x + \sec 5x) \\ &= 5 \sec 5x \end{aligned}$$